

Estimating in-situ state parameter and friction angle in sandy soils from CPT

P.K. Robertson

Gregg Drilling & Testing Inc., Signal Hill, California, USA

ABSTRACT: The response of sandy soils at a constant relative density will vary depending on the effective confining stresses. Research has shown that the state parameter (ψ) is a more meaningful parameter to represent the in-situ state of sandy soil than relative density. Simplified methods have recently been developed to estimate ψ from CPT results based primarily on laboratory test results. Recent methods to evaluate the response of sandy soils to cyclic loading (i.e. liquefaction) base on extensive case histories use an equivalent clean sand normalized cone resistance ($Q_{tn,cs}$) to represent the in-situ state of the soil. This paper illustrates the similarity between the relationships for $Q_{tn,cs}$ and ψ using the updated soil behavior type chart and describes a simplified method to estimate in-situ state parameter and peak friction angle of sandy soils using $Q_{tn,cs}$. The similarity suggests that the clean sand equivalent normalized cone resistance, $Q_{tn,cs}$ is essentially a measure of in-situ state (ψ) for a wide range of soils.

1 INTRODUCTION

Vesic and Clough (1968) showed that the response of sandy soils varies with both relative density (void ratio) and effective stress. At low effective confining stress, dense sands are dilative and strain hardening in undrained shear, whereas, the same dense sand at high effective confining stress can be contractive and strain softening. Bolton (1986) reviewed extensive laboratory data on sands and developed a simplified relationship between relative density, effective confining stress and peak friction angle. Been and Jefferies (1985) combined in-situ void ratio and effective confining stress to develop a state parameter (ψ) to capture the in-situ response of sands and showed that ψ more accurately represents in-situ state for sandy soils. Plewes et al (1992) and Jefferies and Been (2006) developed a simplified method to estimate (ψ) using CPT results based primarily on laboratory and calibration chamber test results. Much of the test results were on clean uniform sands, with few on silty sands.

Robertson and Wride (1998), Moss et al (2006), and others have evaluated liquefaction case histories to develop relationships between normalized cone resistance and the resistance to cyclic loading. The extensive case history database showed that the normalized cone resistance requires modification as the sandy soils increased in fines content. Robertson and Wride (1998) suggested the concept of a clean sand equivalent normalized cone resistance ($Q_{tn,cs}$) that would represent the equivalent clean sand resistance for CPT results in silty sands. Essentially, the clean sand equivalent normalized cone resistance is a simplified means, based on case histories, to represent the in-situ state of sandy soils, over a wide range of soils, from clean sand to silty sands.

The objective of this paper is to evaluate the possible link between state parameter (ψ) and equivalent clean sand normalized cone resistance ($Q_{tn,cs}$) in an effort to develop a simplified method to estimate ψ and peak friction angle (ϕ) for a wide range of sandy soils from CPT results.

2 CLEAN SAND EQUIVALENT CONE RESISTANCE

Robertson (1990) developed a chart to identify Soil Behavior Type (SBT) based on normalized CPT parameters. The CPT parameters are normalized by the effective overburden stress to produce dimensionless parameters, Q_t and F_r , where;

$$Q_t = (q_t - \sigma_{vo}) / \sigma'_{vo} \quad (1)$$

$$F_r = [(f_s / (q_t - \sigma_{vo}))] 100\% \quad (2)$$

Where: q_t = CPT corrected total cone resistance
 f_s = CPT sleeve friction
 σ_{vo} = pre-insertion in-situ total vertical stress
 σ'_{vo} = pre-insertion in-situ effective vertical stress

Jefferies and Davies (1993) identified that a Soil Behavior Type Index, I_c , could represent the SBT zones in the $Q_t - F_r$ chart where, I_c is essentially the radius of concentric circles that define the boundaries of soil type. Robertson and Wride, (1998) modified the definition of I_c to apply to the Robertson (1990) $Q_t - F_r$ chart, as defined by:

$$I_c = [(3.47 - \log Q_t)^2 + (\log F_r + 1.22)^2]^{0.5} \quad (3)$$

Robertson and Wride (1998), as updated by Zhang et al (2002), suggested a normalized cone parameter to evaluate soil liquefaction, using normalization with a variable stress exponent, n ; where:

$$Q_{tn} = [(q_t - \sigma_v) / p_a] (p_a / \sigma'_{vo})^n \quad (4)$$

Where: $(q_t - \sigma_v) / p_a$ = dimensionless net cone resistance, and,
 $(p_a / \sigma'_{vo})^n$ = stress normalization factor

n = stress exponent that varies with SBT
 p_a = atmospheric pressure in same units as q_t and σ_v

Note that when $n = 1$, $Q_{tn} = Q_t$. Zhang et al (2002) suggested that the stress exponent, n , could be estimated using the SBT Index, I_c , and that I_c should be defined using Q_{tn} .

In recent years there have been several publications regarding the appropriate stress normalization (Olsen and Malone, 1988; Zhang et al., 2002; Idriss and Boulanger, 2004; Moss et al, 2006; Cetin and Isik, 2007). The contours of stress exponent suggested by Cetin and Isik (2007) are very similar to those by Zhang et al, (2002). The contours by Moss et al (2006) are similar to those first suggested by Olsen and Malone (1988). The normalization suggested by Idriss and Boulanger (2004) only apply to sands where the stress exponent varies with relative density with a value of around 0.8 in loose sands and 0.3 in dense sands. Robertson (2009) provided a detailed discussion on stress normalization and suggested the following updated approach to allow for a variation of the stress exponent with both SBT I_c (soil type) and stress level using:

$$n = 0.381 (I_c) + 0.05 (\sigma'_{vo}/p_a) - 0.15 \quad (5)$$

where $n \leq 1.0$

Robertson (2009) suggested that the above stress exponent would capture the correct in-situ state for soils at high stress level and that this would also avoid any additional stress level correction for liquefaction analyses.

It is well recognized that the normalized cone resistance decreases as a soil becomes more fine-grained, due to the increasing compressibility of fine-grained soils compared to coarse-grained soils. This was identified by Robertson (1990) where the normally consolidated region on the CPT SBT chart extends down the chart, i.e. as soil becomes more fine-grained the normalized cone resistance (Q_{tn}) decreases and F_r increases. Robertson and Wride (1998) suggested that the soil behavior index, I_c increases when soils become more fine-grained and that when $I_c > 2.60$ soils tend to be more clay-like.

Robertson and Wride (1998), based on a large database of liquefaction case histories, suggested a correction factor to correct normalized cone resistance in silty sands to an equivalent clean sand value ($Q_{tn,cs}$) using the following:

$$Q_{tn,cs} = K_c Q_{tn} \quad (6)$$

where K_c is a correction factor that is a function of grain characteristics (combined influence of fines content, mineralogy and plasticity) of the soil that can be estimated using I_c as follows:

if $I_c \leq 1.64$

$$K_c = 1.0 \quad (7)$$

if $I_c > 1.64$

$$K_c = 5.581 I_c^3 - 0.403 I_c^4 - 21.63 I_c^2 + 33.75 I_c - 17.88 \quad (8)$$

Figure 1 shows contours of equivalent clean sand cone resistance, $Q_{tn,cs}$, on the updated CPT SBT chart. The contours of $Q_{tn,cs}$ were developed based on liquefaction cased histories.

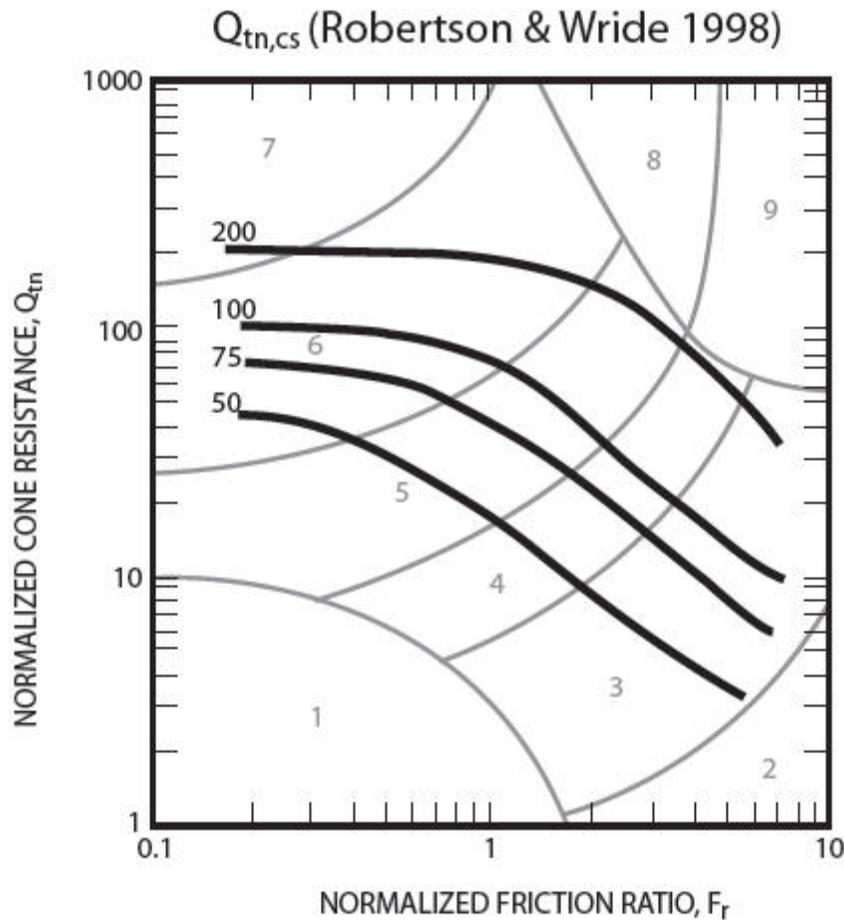


Figure 1 Contours of clean sand equivalent normalized cone resistance based on Robertson & Wride, (1998)

3 STATE PARAMETER (ψ)

The state parameter (ψ) is defined as the difference between the current void ratio, e and the void ratio at critical state e_{cs} , at the same mean effective stress. Based on critical state concepts, Jefferies and Been (2006) provide a detailed description of the evaluation of soil state using the CPT. They describe in detail that the inverse prob-

lem of evaluating state from CPT response is complex and depends on several soil parameters. The main parameters are essentially the shear stiffness, shear strength, compressibility and plastic hardening. Jefferies and Been (2006) provide a description of how state can be evaluated using a combination of laboratory and in-situ tests. They stress the importance of determining the in-situ horizontal effective stress and shear modulus using in-situ tests and determining the shear strength, compressibility and plastic hardening parameters from laboratory testing on reconstituted samples. They also show how the inverse problem can be assisted using numerical modeling. For high risk projects a detailed interpretation of CPT results using laboratory results and numerical modeling can be appropriate (e.g. Shuttle and Cunning, 2007), although soil variability can complicate the interpretation procedure. For low risk projects and in the initial screening for high risk projects there is a need for a simple estimate of soil state. Plewes et al (1992) provided a means to estimate soil state using the normalized SBT chart suggested by Jefferies and Davies (1991). Jefferies and Been (2006) updated this approach using the normalized SBT chart based on the parameter $Q_t(1-B_q) + 1$. Robertson (2009) expressed concerns about the accuracy and precision of the Jefferies and Been (2006) normalized parameter in soft soils. In sands, where $B_q = 0$, the normalization suggested by Jefferies and Been (2006) is the same as Robertson (1990). The contours of state parameter (ψ) suggested by Plewes et al (1992) and Jefferies and Been (2006) were based primarily on calibration chamber results for sands.

Based on the data presented by Jefferies and Been (2006) and Shuttle and Cunning (2007) as well the measurements from the CANLEX project (Wride et al, 2000) for predominately coarse-grained uncemented young soils, combined with the link between OCR and state parameter in fine-grained soil, Robertson (2009) developed contours of state parameter (ψ) on the updated SBTn $Q_{tn} - F$ chart for uncemented Holocene age soils. The contours, that are shown on Figure 2, are approximate since stress state and plastic hardening will also influence the estimate of in-situ soil state in the coarse-grained region of the chart (i.e. when $I_c < 2.60$) and soil sensitivity for fine-grained soils. An area of uncertainty in the approach used by Jefferies and Been (2006) is the use of Q_{t1} rather than Q_{tn} . Figure 2 uses Q_{tn} since it is believed that this form of normalized parameter has wider application, although this issue may not be fully resolved for some time. The contours of ψ shown in Figure 2 were developed primarily on laboratory test results and validated with well documented sites where undisturbed frozen samples were obtained (Wride et al., 2000).

4 PROPOSED CORRELATION BETWEEN ψ AND $Q_{TN,CS}$

Comparing Figures 1 and 2 shows a strong similarity between the contours of ψ and the contours of $Q_{tn,cs}$. Based on Figures 1 and 2, the following simplified and approximate relationship can be developed between ψ and $Q_{tn,cs}$:

$$\psi = 0.56 - 0.33 \log Q_{tn,cs} \quad (9)^*$$

Equation 9 provides a simplified approximate way to estimate in-situ state parameter for a wide range of sandy soils.

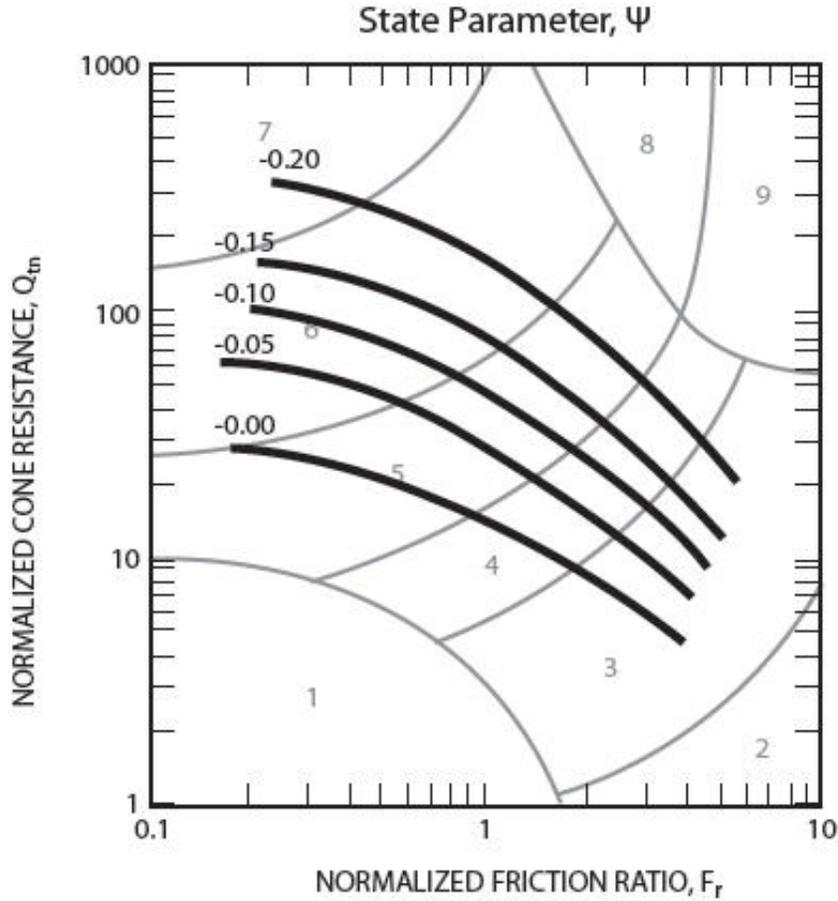


Figure 2 Contours of state parameter (after Robertson, 2009)

5 PROPOSED CORRELATION BETWEEN FRICTION ANGLE, ϕ AND $Q_{TN,CS}$

Jefferies and Been (2006) showed a strong link between ψ and the peak friction angle (ϕ) for a wide range of sands. Using this link, it is possible to link $Q_{tn,cs}$ with ϕ , using:

$$\phi = \phi_{cv} - 48 \psi \tag{10}$$

Where ϕ_{cv} = constant volume (or critical state) friction angle depending on mineralogy (Bolton, 1986), typically about 33 degrees for quartz sands but can be as high as 40 degrees for felspathic sand. Hence, the relationship between $Q_{tn,cs}$ and ϕ becomes:

$$\phi = \phi_{cv} + 15.84 [\log Q_{tn,cs}] - 26.88 \quad (11)*$$

Equation 11 produces estimates of peak friction angle in clean quartz sands that are similar to those by Kulhawy and Mayne (1990). Equation 11 has the advantage that it includes the importance of grain characteristics and mineralogy that are reflected in both ϕ_{cv} , as well as soil type through $Q_{tn,cs}$. Equation 11 tends to predict ϕ values closer to measured values in calcareous sands where the CPT tip resistance can be low for high values of ϕ .

6 SUMMARY

Research has shown that the state parameter (ψ) is a more meaningful parameter to represent the in-situ state of sandy soil than relative density. Simplified methods have been developed to estimate ψ from CPT results based primarily on laboratory test results. Recent methods to evaluate the response of sandy soils to cyclic loading (i.e. liquefaction) based on extensive case histories use an equivalent clean sand normalized cone resistance ($Q_{tn,cs}$) to represent the in-situ state of the soil. This paper has illustrated the similarity between contours of $Q_{tn,cs}$ and contours of ψ on the updated soil behavior type chart and presents a simplified method to estimate the in-situ state parameter of sandy soils using $Q_{tn,cs}$. The similarity suggests that the clean sand equivalent normalized cone resistance $Q_{tn,cs}$ is essentially a measure of in-situ state (ψ) for a wide range of soils.

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